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OSCILLOGRAPHER

CATHODE-RAY Q-METER

by Rudolf Feldt

One of the applications of the cathode-ray oscillograph that is of considerable interest is its use in measuring logarithmic decrement and Q. This method of plotting damped oscillations as fixed images on the cathode-ray tube has both electrical and mechanical significance as will be explained.

THEORY

Any system which is able to oscillate will generate damped oscillations if excited by shock. The frequency of these oscillations is determined by the inductance and capacity of the system and the damping of the oscillations by the losses in the circuit. A damped oscillation is expressed by the formula

$$A_t = A_o e^{-\alpha t} \cos \omega t \qquad (1)$$

where A_t is the voltage across the circuit at the time t, A_o the voltage at the moment of excitation t_o, $\omega = 2\pi f$ where f is the natural frequency of the circuit and α is the decrement factor and is given by the equation

$$\alpha = \frac{R}{2L} \qquad (2)$$

where R is the total series resistance of the circuit and L its inductance.

The logarithmic decrement δ is defined as the natural logarithm of the ratio of the maximum voltage during any given oscillation to the smaller maximum voltage during the next oscillation, one cycle later. It is readily shown that the logarithmic decrement is also equal to the decrement factor divided by the natural frequency. The proof follows:

Referring to Figure 1, the ratio of the voltage at one instant t_o, to that at a later instant t, can be written by use of equation (1)

$$\frac{A_o e^{-\alpha t_o} \cos \omega t_o}{A_1 e^{-\alpha t_1} \cos \omega t_1}$$

If t_o and t_1 are exactly one cycle apart so that

$$\omega t_1 = \omega t_o + 2\pi$$

the cosine in the numerator will equal the cosine in the denominator, and the ratio becomes

$$\frac{e^{-\alpha t_o}}{e^{-\alpha t_1}}$$

The natural logarithm of this ratio is

$$\ln e^{-\alpha t_o} - \ln e^{-\alpha t_1}$$
$$= -\alpha t_o + \alpha t_1$$
$$= \alpha (t_1 - t_o)$$

The interval of time $(t_1 - t_o)$ is the duration of one cycle; it is therefore equal to the reciprocal of the natural frequency. Therefore

$$\alpha (t_1 - t_o) = \alpha / f \qquad (3)$$

Since the ratio of the voltages is computed for any values of voltage separated by a time interval of exactly one cycle, it will be the ratio of any voltage crest to the next crest of the same polarity. But the logarithm of this ratio is, by definition, the logarithmic decrement, so —

Logarithmic decrement $\delta = \alpha/f$

Applying equation (2)

$$\delta = R/2Lf = \ln\frac{A_o}{A_1} \qquad (4)$$

The logarithmic decrement may also be expressed as the ratio between the first and nth maximum amplitude of the damped oscillation, thus:

$$\delta = \frac{R}{2Lf} = \frac{1}{n}\ln\frac{A_o}{A_n} \qquad (5)$$

A very simple relation exists between δ and Q, so

$$\frac{\delta}{\pi} = \frac{R}{2\pi fL} = \frac{R}{\omega L} = \frac{1}{Q}$$

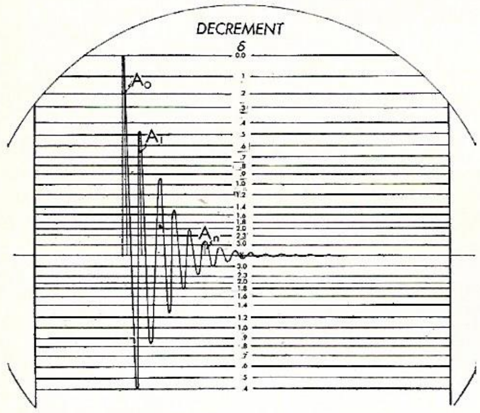

Fig. 1

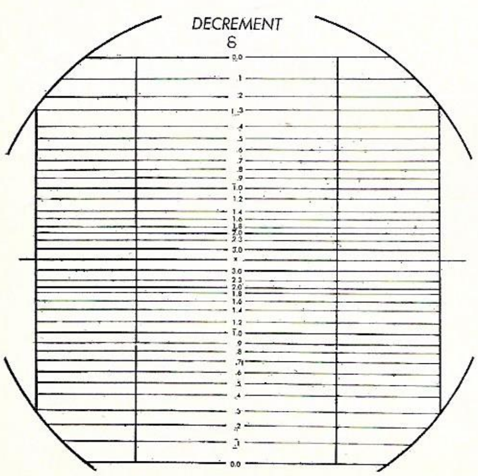

Fig. 2

Therefore

$$Q = \frac{\pi}{\delta} = \frac{R\pi}{\ln\frac{A_o}{A_n}} \qquad (6)$$

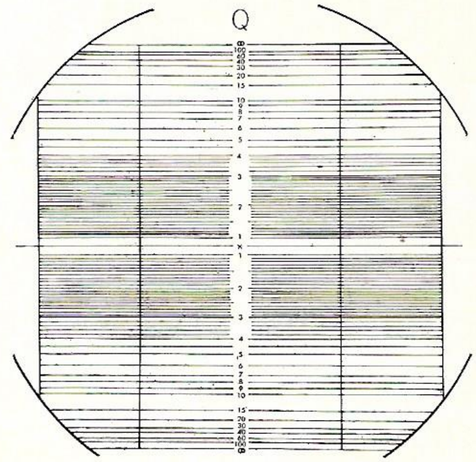

Fig. 3

MEASUREMENT

Shock excitation of tuned circuits may be obtained by applying square waves of suitable frequency to the circuit. A simpler way is to obtain repetitive sharp pulses by differentiating the saw tooth wave of the sweep generator which is part of almost every cathode-ray oscillograph. These pulses are then fed into the oscillatory circuit, and the damped oscillations which are generated are applied to the vertical amplifier terminals of the cathode-ray oscillograph, while the sweep frequency is applied to the horizontal plates in the usual manner. The sweep frequency from which the exciting pulse is derived must be lower than the natural frequency of the tuned circuit in order to produce oscillations which will be visible as stationary images on the screen of the cathode-ray tube.

Numerical values of the decrement and of Q can easily be obtained from the pattern of the damped oscillation by means of transparent scales placed in front of the cathode-ray tube screen. These scales are calibrated logarithmically for δ according to equation (5) and for Q according to equation (6).

Figure 2 shows such a scale calibrated for logarithmic decrement δ and Figure 3 shows a scale for Q.

The procedure of measurement is as follows: The pattern of the damped oscillation is centered with respect to the center line marked X on the scale. The amplitude of the oscillation must be such that one of the peaks touches the zero reference line which corresponds to a deflection of two inches. The amplitude of the next following peak then indicates on the scale (Figure 2), the value of the logarithmic decrement δ. The accuracy can be increased n times by taking the reading for the nth peak and dividing the result by n. Figure 1 illustrates the method on the decrement scale. The decrement of these oscillations is approximately 0.5.

The factor Q may be determined in the same manner using the scale shown in Figure 3. In this case the reference line is marked ∞. The pattern is adjusted in such a way that one of the amplitudes of the damped oscillation touches the reference line. Q can then be read on the scale for the nth following peak, multiplying the value on the screen by n to obtain the final result. In both cases the scales are direct reading if $n = 1$.

For accurate measurements, certain precautions must be taken to avoid damping of the circuits to be measured by the pulse generator and the cathode-ray oscillograph. This difficulty can be overcome easily by the use of a cathode follower stage between the saw tooth generator and the circuit, and by employing high impedance input circuits on the oscillograph or by connecting directly to the deflecting plates of the cathode-ray tube.

The following figures show some examples of results obtained by this method. Figure 4 shows the tank coil of a R.F. power supply transformer tuned to the frequency of the secondary winding. The decrement in this case is about 0.1. A peaking coil with a decrement of about 0.6 is pictured in Figure 5. Figure 6 is the same coil illustrated in Figure 5, but with damping by an external resistance. Figure 7 shows Figure 6 superimposed on Figure 5 for comparison. Figure 8 illustrates the behavior of a detuned 455 kc transformer. It presents the characteristics of a band filter. The different amount of damping for the two frequencies of the filter can be clearly seen on the picture. Figure 9 shows the voltage across the voice coil of a loud speaker excited by a square wave of 15 cps, the primary winding being open. From this picture the damping of the loud speaker membrane and its resonant frequency can be determined.

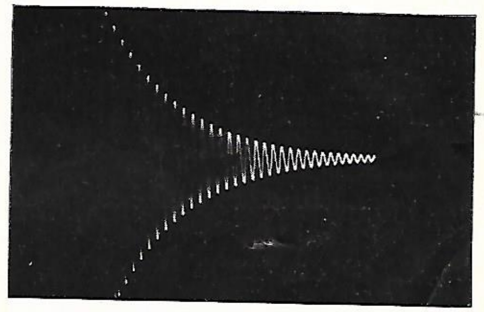

Fig. 4

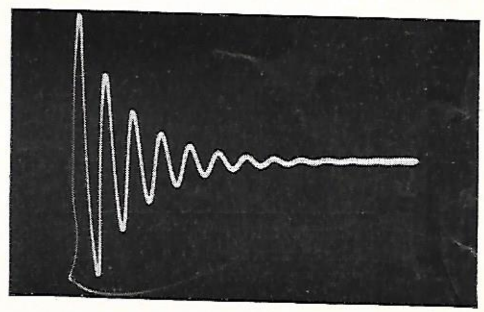

Fig. 5

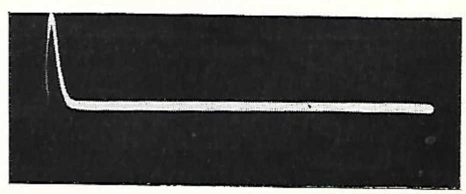

Fig. 6

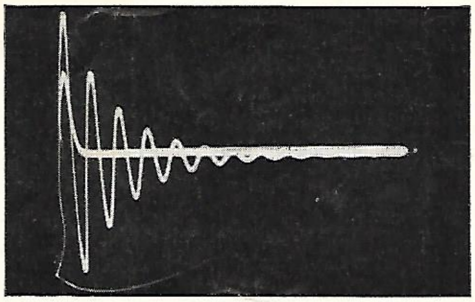

Fig. 7

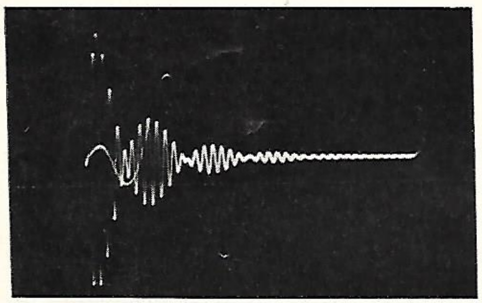

Fig. 8

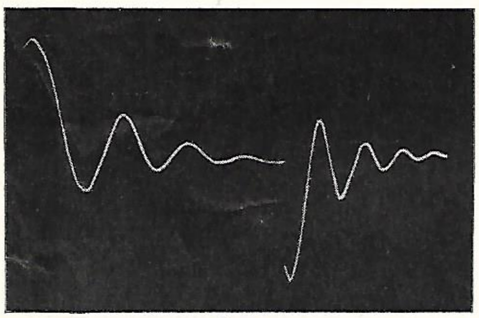

Fig. 9

MECHANICAL APPLICATIONS

This method is readily adapted to measurements on mechanical devices. Any mechanical device which is able to produce vibrations can be excited by shock. This shock may be single or repetitive and will be followed by a series of damped vibrations. If the shock can be synchronized with the sweep circuits, the problem is in no way different from what has been described. The mechanical vibrations can be transformed into electrical voltages by means of piezo electric pickups or by electromagnetic and photoelectric devices, etc. The natural frequency of springs, automobiles and any other kind of machinery can easily be determined in this manner. In cases where the natural frequency is very low, tubes with long persistence screens should be used.

The rapid solution of many other mechanical problems has been made possible by the development of the cathode-ray oscillograph. Complex mechanical problems may be solved by the design of simple electrical analogues. The electrical characteristics of the analogy may then be plotted on the cathode-ray oscillograph and the results may be interpreted in the terms of mechanical characteristics.

A more complete description of the design and use of electrical analogy may be found in an article by E. N. Kemler entitled "Models and Analogues for Solving Design Problems" which appeared in the November 1942 issue of Product Engineering.

Erratum: In the Jan.-Feb., 1945 *issue of the Oscillographer on page* 2, *the second sentence in the section titled "EQUIPMENT" reading "Any oscillograph containing a X-axis amplifier . . ." should read "Any oscillograph containing a Z-axis amplifier . . ."*